

February 1, 2008

UPR-0633T

Four-Fermi Operators in $e^+ e^-$ Annihilation Experiments and Uncertainties in Z Boson Properties

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Abstract

We investigate the effects of possible new four-Fermi operators on $e^+ e^-$ annihilation experiments. They represent a class of new physics, which has the potential to change the parameters of the Z boson and at the same time interferes with their determination from the data. We show that in the presence of such operators the Z parameters obtained from lineshape fits can change significantly. Another important property of these operators is that they spoil the factorization of the expression for the left-right asymmetry, A_{LR} , into initial and final state couplings. Factorization and subsequent cancellation of the final state coupling occurs in the Standard Model (after correcting for photonic amplitudes) and is crucial for the interpretation of A_{LR} as a measurement of the effective weak mixing angle. Four-Fermi operators may thus provide an explanation for the high value of the polarization asymmetry as observed at SLC. However, the data from lower energy $e^+ e^-$ annihilation severely constrains this class of operators and virtually closes this possible loophole on how new physics might explain the SLC/LEP discrepancy. We point out that if the surplus of observed b -pairs at LEP is real and caused by these operators, there may be a significant effect on the forward-backward asymmetry into b -quarks. It is this quantity which presently gives the most precise determination of the weak mixing angle. We present compact analytical expressions for the treatment of initial state radiation in the presence of new contact operators.

I. INTRODUCTION:

As the Z factories LEP 1 and SLC increase their event samples and decrease systematic uncertainties, many quantities related to the Z boson are now known with an impressive accuracy [1]. A recent global analysis [2] shows that the data are generally in impressive agreement with the minimal Standard Model, though some observables show deviations at the 2 or 3 σ level and it cannot be excluded that some of them are due to new physics.

At the heart of high precision experiments are the energy scans around the Z peak which are analyzed using various degrees of model independence. One may work entirely in the context of the Standard Model, using as free parameters the masses of the Z and Higgs bosons and the top quark in addition to the couplings $\alpha(M_Z)$ and $\alpha_s(M_Z)$. Alternatively, one may allow for general vector and axial-vector couplings of the Z to fermions. Sometimes also more general interference terms are admitted¹. All these approaches assume, however, that exclusively Z and photon exchange diagrams contribute to cross sections and asymmetries.

In this paper, we study the possibility that there may be additional contributions to the cross sections (asymmetries) arising through new effective four-Fermi operators. The Z lineshape is known to be well described by a Breit-Wigner curve which is only distorted by photonic contributions and electroweak loop corrections with very little s -dependence. The χ^2 values of the Z lineshape fits are satisfactory and by themselves do not call for the introduction of new parameters beyond some minimal set. However, a major motivation for high precision experiments is the hope to uncover new physics beyond the Standard Model. Likewise, one wishes to extract limits on new physics, such as additional Z 's or compositeness. This is usually done by comparing the observed Z properties with the Standard Model expectations and with expectations from certain kinds of new physics. But there is the logical possibility that new interactions contribute to the event samples at $e^+ e^-$ annihilation experiments and obscure the Z studies. If new effects are present but not corrected for one would extract erroneous parameters from the data.

Presently, the most important motivation for discussing new four-Fermi interactions is the anomalously high left-right asymmetry, A_{LR} , as observed by SLD [5]. We showed in ref. [6] that even the most general choice of Z couplings cannot significantly decrease the apparent discrepancy between A_{LR} and other high precision observables, and in particular to the LEP asymmetries. Moreover, using results from reference [7] it was argued [6] that no

¹See ref. [3] for a description of the new ZFITTER [4] interface SMATASY and review of other approaches.

kind of new physics can account for the SLD result without simultaneously conflicting with one or several other observables, most notably the W mass. Again, the one loophole in such a line of argument could be a new interaction which significantly contributes to the observed four fermion processes and so influences the extracted Z parameters. In the absence of new operators and after correcting for photonic contributions, the asymmetries factorize into expressions describing initial and final state couplings (see Eqs. (7) – (10) below). In the case of A_{LR} the final state couplings then cancel and one is left with an expression containing just the effective weak mixing angle for electrons. On the other hand, in the presence of new operators factorization and cancellation cease to hold so that the observables at LEP and SLC describe inequivalent quantities.

At first glance it seems unlikely that there is any kind of new physics which contributes significantly to A_{LR} without simultaneously affecting the high statistic LEP cross section measurements in an unacceptable way. Surprisingly, as we discuss in section III, Z lineshape data alone cannot exclude a sizable contribution from new physics. It is the data from lower energy $e^+ e^-$ annihilation experiments which severely constrain this class of operators and virtually close a possible loophole on how new physics might explain the SLC result. There are no Z interferences on the pole, but they arise away from the pole and there they effect the factorization. Far away from the Z pole the interference of the new physics with the photon is in general unsuppressed and this yields strong constraints for vector operators.

In section II we collect some basic facts about effective four-Fermi operators and discuss in detail (pseudo-) scalar and tensor operators. Section III focuses on (axial-) vector operators and on how they might affect the lineshape measurements. For our analyses we use the published cross section data from L3 [8]. In section IV we summarize our conclusions. Appendix A describes the approximations we used for our fits. Explicit formulae for initial state radiation in the presence of four-Fermi operators are presented in appendix B.

II. FOUR-FERMI OPERATORS AND HELICITY AMPLITUDES:

The most general four-fermion contact operator has the form [9]

$$\mathcal{L}_{\text{eff}} = -\frac{4\pi}{\Lambda^2} \sum_{i=S,P,V,A,T} \bar{f}_1 \Gamma^i f_2 (D_i^{f_1,f_2,f_3,f_4} \bar{f}_3 \Gamma_i f_4 + \tilde{D}_i^{f_1,f_2,f_3,f_4} \bar{f}_3 \Gamma_i \gamma_5 f_4) + h. c., \quad (1)$$

where the sum is over scalar, pseudo-scalar, vector, axial-vector and tensor operators. The scale Λ is introduced for convenience and later it will be taken to be 1 TeV. For $e^+ e^-$ annihilation we set $f_1 = f_2 = e^-$. For definiteness we restrict ourselves to flavor conserving

neutral currents² so that $f_3 = f_4$. Then the effective Lagrangian (1) can be rewritten as

$$\mathcal{L}_{\text{eff}} = -\frac{4\pi}{\Lambda^2} \sum_{i,j=L,R} (S_{ij}^f \bar{e}^- P_i e^- \bar{f} P_j f + V_{ij}^f \bar{e}^- \gamma^\mu P_i e^- \bar{f} \gamma_\mu P_j f + T_{ij}^f \bar{e}^- \frac{\sigma^{\mu\nu}}{\sqrt{2}} P_i e^- \bar{f} \frac{\sigma_{\mu\nu}}{\sqrt{2}} P_j f) + h.c., \quad (2)$$

where $P_i = (1 + h_i \gamma_5)/2$ with $h_i = \mp 1$ for left and right-handed fermions, respectively. The coefficients S_{ij}^f and T_{ij}^f satisfy the relations

$$\begin{aligned} S_{LL}^f &= S_{RR}^{f*}, \\ S_{LR}^f &= S_{RL}^{f*}, \\ T_{LL}^f &= T_{RR}^{f*}, \\ T_{LR}^f &= T_{RL}^f = 0, \end{aligned} \quad (3)$$

whereas the V_{ij}^f are real³.

The helicity amplitudes for $e^+ e^-$ annihilation through (axial-) vector channels are given by

$$\begin{aligned} \frac{d\sigma_{ijf}^V}{dz}(s) &= \frac{\pi N_c s}{8} \left| \alpha \frac{g_i^e g_j^f}{s - M_Z^2 + i M_Z \Gamma_Z} + \alpha \frac{Q_e Q_f}{s} + \frac{V_{ij}^f}{\Lambda^2} \right|^2 (1 + h_i h_j z)^2 \\ &\equiv \frac{\pi N_c s}{8} |A_{ij}^f|^2 (1 + h_i h_j z)^2, \end{aligned} \quad (4)$$

where $z = \cos \theta$. The first term of the amplitude is the Z contribution, where g_i^f describes the effective coupling of the Z to fermion f with helicity h_i . The second term is the contribution from QED and the third one from new physics.

If we now define cross sections for a given fermion flavor of specific helicity to be detected in the forward or backward hemisphere,

$$\begin{aligned} \sigma_{ijf}^{V^F} &\sim \int_0^1 dz [1 + h_i h_j z]^2, \\ \sigma_{ijf}^{V^B} &\sim \int_{-1}^0 dz [1 + h_i h_j z]^2, \end{aligned} \quad (5)$$

we find for forward-backward asymmetries

$$A_{FB}(i, j, f) \equiv \frac{\sigma_{ijf}^F - \sigma_{ijf}^B}{\sigma_{ijf}^F + \sigma_{ijf}^B} = \frac{3}{4} h_i h_j. \quad (6)$$

²We expect our conclusions to hold for the case $f_3 \neq f_4$, as well.

³On the pole interferences with the Z are only possible for complex couplings, but these are not allowed for $f_3 = f_4$. If $f_3 \neq f_4$ there is no interference due to the absence of flavor changing neutral currents in the Standard Model.

Averaging over initial and summing over final helicity states yields

$$A_{FB}(f) = \frac{3}{4} \frac{|A_{LL}^f|^2 + |A_{RR}^f|^2 - |A_{LR}^f|^2 - |A_{RL}^f|^2}{|A_{LL}^f|^2 + |A_{RR}^f|^2 + |A_{LR}^f|^2 + |A_{RL}^f|^2} = \frac{3}{4} \frac{(g_L^{e^2} - g_R^{e^2})(g_L^{f^2} - g_R^{f^2})}{(g_L^{e^2} + g_R^{e^2})(g_L^{f^2} + g_R^{f^2})} \equiv \frac{3}{4} A_e A_f, \quad (7)$$

where the second equality holds when one ignores any photonic or new physics contributions, in which case the result is independent of s . All s -dependence enters through the photon amplitude and possibly through new interactions.

Similarly, one has for the left-right asymmetry

$$A_{LR}(f) = \frac{|A_{LL}^f|^2 + |A_{LR}^f|^2 - |A_{RL}^f|^2 - |A_{RR}^f|^2}{|A_{LL}^f|^2 + |A_{LR}^f|^2 + |A_{RL}^f|^2 + |A_{RR}^f|^2} = \frac{(g_L^{e^2} - g_R^{e^2})(g_L^{f^2} + g_R^{f^2})}{(g_L^{e^2} + g_R^{e^2})(g_L^{f^2} + g_R^{f^2})} = A_e, \quad (8)$$

and it becomes obvious that the final state couplings drop out even after summation over final state fermion flavors⁴. After inclusion of photonic or new physics amplitudes the factorization and cancellation of the final state couplings ceases to hold.

The τ -polarization and its forward-backward asymmetry are given by

$$\mathcal{P}_\tau = \frac{|A_{RR}^\tau|^2 + |A_{LR}^\tau|^2 - |A_{RL}^\tau|^2 - |A_{LL}^\tau|^2}{|A_{RR}^\tau|^2 + |A_{LR}^\tau|^2 + |A_{RL}^\tau|^2 + |A_{LL}^\tau|^2} = \frac{g_R^{\tau^2} - g_L^{\tau^2}}{g_R^{\tau^2} + g_L^{\tau^2}} = -A_\tau, \quad (9)$$

and

$$\mathcal{P}_\tau^{FB} = \frac{3}{4} \frac{|A_{RR}^\tau|^2 - |A_{LR}^\tau|^2 + |A_{RL}^\tau|^2 - |A_{LL}^\tau|^2}{|A_{RR}^\tau|^2 + |A_{LR}^\tau|^2 + |A_{RL}^\tau|^2 + |A_{LL}^\tau|^2} = -\frac{3}{4} \frac{g_R^{e^2} - g_L^{e^2}}{g_R^{e^2} + g_L^{e^2}} = -\frac{3}{4} A_e, \quad (10)$$

respectively. Thus, after correcting for all photonic effects (including initial state radiation) $A_{LR}(\tau)$ and \mathcal{P}_τ^{FB} are physically equivalent quantities. Moreover, even in the presence of photonic and/or new physics contributions to the amplitude they are identical functions⁵ of s . In practice, however, less than 1% of the SLD sample consists of τ -pairs [5].

Let us compare this with the case of (pseudo-) scalar interactions for which we find

$$\frac{d\sigma_{ijf}^S}{dz}(s) = \frac{\pi N_c s}{8} |S_{ij}^f|^2. \quad (11)$$

Notice, that this result is independent of the scattering angle θ as well as the helicity structure. The first property trivially shows that all forward-backward asymmetries vanish for this class of operators. The second property combined with the hermiticity conditions (3)

⁴SLD counts hadronic and τ -events.

⁵They receive slightly different corrections from initial state radiation, though.

for the S_{ij}^f shows that the left-right asymmetries vanish as well⁶. On the other hand, such operators contribute to total cross sections and consequently they can lower the asymmetries from (axial-) vector type interactions.

Is it possible that LEP asymmetries are lowered more significantly in this way than the SLD asymmetries? Presently, the most precise determination of the weak mixing angle comes from $A_{FB}(b)$, which corresponds to

$$A_e[A_{FB}(b)] = 0.1380 \pm 0.0051, \quad (12)$$

and which is to be compared with

$$A_e[A_{LR}] = 0.1637 \pm 0.0075. \quad (13)$$

Thus the two most precise determinations show the largest discrepancy (2.8σ). So let us assume that there is an additional contribution to the cross section into b -quarks effectively lowering $A_{FB}(b)$ such as to resolve the discrepancy. Neglecting the τ -pairs at SLC we find for the fraction of b -events due to new physics

$$\frac{\sigma_b^{\text{new}}}{\sigma_b} = \frac{A_e[A_{LR}] - A_e[A_{FB}(b)]}{A_e[A_{LR}] - R_b A_e[A_{FB}(b)]} \approx 0.19, \quad (14)$$

where we used the LEP value $R_b = \sigma_b/\sigma_{had} = 0.2192 \pm 0.0018$ for R_c fixed to its Standard Model value. This would correspond to

$$A_e^{\text{SM}} = \frac{A_e[A_{FB}(b)]}{1 - \frac{\sigma_b^{\text{new}}}{\sigma_b}} = \frac{A_e[A_{LR}]}{1 - R_b \frac{\sigma_b^{\text{new}}}{\sigma_b}} \approx 0.171, \quad (15)$$

and to $m_t \approx 270$ GeV (for $M_H = 300$ GeV). We use this value of m_t to compute

$$R_b = \frac{R_b^{\text{SM}}}{1 + (R_b^{\text{SM}} - 1) \frac{\sigma_b^{\text{new}}}{\sigma_b}} \approx 0.249, \quad (16)$$

which is in clear conflict with the value from LEP. In any case, such a high value of m_t would also be in sharp conflict with other observables, such as the W mass and, of course, with the top-quark interpretation of the CDF events.

In conclusion, (pseudo-) scalar four-Fermi operators cannot resolve the LEP/SLC discrepancy and have little impact on the asymmetries. Still, it is interesting to note that in

⁶The only non-trivial asymmetry would be a combined initial and final state polarization asymmetry, i.e., a measurement of final state helicities using a polarized beam. This kind of asymmetry has not been measured, yet.

the presence of new physics which produces additional b -quarks (1) the LEP/SLC discrepancy becomes slightly smaller, (2) the R_b measurement is accounted for, and (3) the high values for α_s from R_{had} are lowered [2] to be in better agreement with most low energy determinations.

The next class of effective operators to be discussed are of (pseudo-) tensor type. In this case we find for the cross sections,

$$\frac{d\sigma_{ijf}^T}{dz}(s) = \frac{\pi N_c s}{4} \left| \frac{T_{ij}^f}{\Lambda^2} \right|^2 (1 + h_i h_j) z^2. \quad (17)$$

Since the cross sections are proportional to $\cos^2 \theta$ there is again no forward-backward asymmetry for this class of operators. In accordance with the last relation (3) we see that helicity changing amplitudes vanish. The hermiticity condition on the coefficients for helicity conserving processes implies the vanishing of A_{LR} , as well. Thus the same conclusions apply for (pseudo-) tensor operators as for the (pseudo-) scalar class.

The situation changes when both types of operators are present simultaneously. In this case interference terms are possible for helicity conserving processes,

$$\frac{d\sigma_{ijf}^{S+T,I}}{dz}(s) = -\frac{\pi N_c s}{4\Lambda^4} \text{Re} (S_{ij}^f T_{ij}^{f*}) (1 + h_i h_j) z. \quad (18)$$

Since these terms are linear in $\cos \theta$, we find non-trivial contributions to forward-backward asymmetries if the coefficients are not out of phase. The cross section for combined scalar and tensor interactions is

$$\frac{d\sigma_{ijf}^{S+T}}{dz}(s) = \frac{\pi N_c s}{8\Lambda^4} |S_{ij}^f - T_{ij}^f (1 + h_i h_j) z|^2. \quad (19)$$

From this expression it can again be shown that A_{LR} vanishes identically. The (unnormalized) left-right asymmetry, $\sigma_L - \sigma_R$, is sensitive precisely to the (axial-) vector part of the theory. The forward-backward asymmetry, on the other hand, takes on the form

$$A_{FB}^{S+T} = \frac{-2\text{Re} (S_{LL}^f T_{LL}^{f*})}{|S_{LL}^f|^2 + |S_{LR}^f|^2 + \frac{4}{3}|T_{LL}^f|^2}. \quad (20)$$

It takes its maximal negative value for $S_{LR}^f = 0$ and $T_{LL}^f = \frac{\sqrt{3}}{2} S_{LL}^f$, in which case it is

$$A_{FB}^{S+T,\max} = -\frac{\sqrt{3}}{2} \approx -86.6\%. \quad (21)$$

It should be noted that the new physics contributions considered here always spoil the factorization properties (second equal signs in Eqs. (7) – (10)) trivially in that they contribute

to the total cross section in the denominators. But here the factorization is affected in addition through a direct contribution to $\sigma^F - \sigma^B$.

We have to ask again, whether it is possible that LEP asymmetries are changed more significantly than A_{LR} . Suppose that $\frac{\sigma_b^{\text{new}}}{\sigma_b} \approx 3.3\%$. Then using Eq. (15) yields $A_e^{\text{SM}} \approx 0.165$. This corresponds to $m_t = 254$ GeV (again for $M_H = 300$ GeV) and in this case Eq. (16) is satisfied for the LEP value of R_b . The forward-backward asymmetry into b -quarks would now be given by the cross section weighted average

$$A_{FB}(b) = (1 - \frac{\sigma_b^{\text{new}}}{\sigma_b})A_{FB}^{SM}(b) + \frac{\sigma_b^{\text{new}}}{\sigma_b}A_{FB}^{S+T}(b). \quad (22)$$

Using $A_{FB}(b) = 0.0967 \pm 0.0038$ from LEP and $A_{FB}^{SM}(b) = 0.1157$ we find $A_{FB}^{S+T}(b) \approx -46\%$. This is clearly acceptable, but there remains the conflict between A_{LR} and A_e from \mathcal{P}_τ^{FB} which would now be about 2.3σ . The point is that final state asymmetries are as insensitive to these operators as initial state asymmetries. In order to solve this conflict, one would have to assume that about 18% of the cross section into τ -pairs is due to new physics. This would be in sharp conflict with the $Z \rightarrow \tau^+\tau^-$ width, which is determined to be in excellent agreement with the Standard Model value. Including A_τ from \mathcal{P}_τ increases this discrepancy to 2.5σ ; still an admixture of 16% τ -pairs from new physics would be required. Even if one assumes that the τ -polarization measurements are principally wrong and dismisses them, one still faces the above mentioned conflicts between a high m_t on one hand and M_W and CDF on the other.

In summary, we have shown that the data do not support the hypothesis that (pseudo) scalar and tensor four-Fermi operators may resolve the discrepancy of A_{LR} with other asymmetries. These operators do not contribute to polarization asymmetries and can at best resolve conflicts between forward-backward asymmetries and polarization asymmetries, but not between different polarization asymmetries. Moreover, as shown they can only lower the measured asymmetries and hence increase the extracted bare asymmetries. This would lead to very high values of m_t which are inconsistent with many other observables. On the other hand, if the excess of b -quarks over the Standard Model value as measured at LEP is real and due to these operators, the extracted value of $\sin^2 \theta_{\text{eff}}^e$ from $A_{FB}(b)$ could be significantly lower or higher. This is particularly interesting in view of the fact that by now $A_{FB}(b)$ yields the most precise determination of the weak mixing angle.

III. VECTOR AND AXIAL-VECTOR OPERATORS:

In the previous section we discussed the four helicity amplitudes related to (pseudo-) scalar and tensor operators. They are the ones describing interactions between states of opposite helicities. There are four other amplitudes for interactions between states of equal helicities. There cannot be interferences between the eight different helicity structures. The new features related to (axial-) vector operators are that they can interfere with the photon and the Z and that they lead to non-trivial polarization asymmetries. Also, they are theoretically better motivated. E.g., they may arise from heavy Z' bosons, whose presence would not spoil the successful supersymmetric gauge coupling unification. In that case new effects can show up in two different ways: (1) $Z - Z'$ mixing can change the mass and coupling relations of the ordinary Z . (2) The determination of Z properties from the data is modified.

It is often believed that such new operators cannot significantly contribute to the high statistics and high precision measurements at the Z pole since the Z lineshape is measured so well. But the approximate Breit-Wigner curve only proves that the Z dominates the measurements. Other contributions could still affect the details of the fit (parameters). It is also not appropriate to conclude from the decent χ^2 values from the lineshape fits that there cannot be significant contributions from new sources. It is amusing to note that for the L3 cross section data χ^2 actually decreases by 0.8 when photon exchange and interference are omitted⁷! In such a fit, the Z width and the leptonic partial width increase by about 10 and 1 MeV, respectively, whereas the hadronic bare cross section, σ_{had}^0 , and M_Z remain unchanged. This exercise should warn us that new physics which couples as strong as the photon or even stronger could have been overlooked and significantly influence lineshape parameters.

Suppose now that there are new purely vector-like four-Fermi operators for hadrons only ($V_{ij}^q = V^q \neq 0$, $V_{ij}^l = 0$). This is a particularly interesting case for the following reasons:

1. The anomalously high 1993 data at SLC was taken at $\sqrt{s} = 91.26$ GeV, whereas the low statistics run of 1992 at $\sqrt{s} = 91.55$ GeV [10] yielded a 1.5σ lower result for A_{LR} . At first sight this does not seem to be a possible hint that we see some (additional) s -dependence from interference terms. It seems that the critical 1993 measurement was closer to M_Z than the 1992 run and hence that new interferences would be more

⁷In order to avoid complications with t -channel exchange we omit the data for final state electrons.

strongly suppressed in 1993. However, initial state radiation effects effectively shift the peak position by more than 200 MeV to higher energies, so that interference effects in 1993 are larger and in 1992 smaller than naively expected.

2. The interference terms with the Z contributing to A_{LR} are proportional to the axial-vector coupling, g_A^e , whereas the ones contributing to total cross sections are suppressed by the small vector coupling, g_V^e .
3. We have to require destructive interference of the new physics with the photon in order to avoid too large contributions to the hadronic cross section; this fixes the sign of V^q to be the same as the sign of Q_q . With this choice the Z interference effect which might explain the high A_{LR} also has the right sign.
4. We included these operators into a Z lineshape fit using L3 cross section data and assuming family universality ($V^u = V^c$ and $V^d = V^s = V^b$) we found a χ^2 minimum for $V^u \sim V^d \sim 0.5$, although the result was consistent with $V^q = 0$. Note, that the values $V^u = 0.585$ or $V^d = 0.293$ correspond to an interaction strength comparable to QED.
5. The combined fit of the L3 and SLC data gives a significant χ^2 minimum at $V^u = 1.2 \pm 1.0$ and $V^d = -1.2 \pm 0.6$. Requiring a maximal value for A_{LR} gives virtually the same answer, showing that cross section measurements around the Z pole are not very sensitive to these new operators and that the fit result is dominated by the SLD asymmetry data.

The maximal effect for A_{LR} comes about because for higher values of the V^q the contribution to the total hadronic cross section would overcompensate the interference effect. We found shifts of ± 8 MeV in M_Z and Γ_Z , respectively, whereas σ_{had}^0 and Γ_l decreased by less than a standard deviation.

Allowing these new operators increases the theory value of A_{LR} (91.26 GeV) from 0.1385 to 0.1469, which is still low compared to the measured value of 0.1656. About a third of the discrepancy could be accounted for, but it is still 2.1σ . However, it is the older data from $e^+ e^-$ annihilation experiments well below the Z which eliminates this scenario. At the Z peak the new physics would be stronger than QED. Because of the destructive interference with the photon one would predict that at some energy below the Z the combined amplitude from QED and the new physics would vanish. At $\sqrt{s} = 34$ GeV, QED would be stronger than the new physics but still one would expect only about half of the observed hadronic cross section. What has originally been seen was actually a slight enhancement of the cross

section over the Standard Model prediction, but a new analysis by Haidt [11] now shows agreement.

Henceforth, we will assume that new physics interference with the photon vanishes, i.e., we require

$$V_{LL}^f + V_{LR}^f + V_{RL}^f + V_{RR}^f = 0. \quad (23)$$

In the remainder of this section we have a closer look at two representative cases. The first case (a) focusses on the possibility that A_{LR} may be enhanced by interference effects, like for the pure vector operators discussed before. The second case (b) assumes that the new physics couples to left-handed electrons only, which obviously would enhance A_{LR} , as well.

Since case (a) concentrates on interference terms to A_{LR} we choose $V_{LL}^f = V_{RL}^f$ in order to retain the enhancement of g_A^e/g_V^e compared to the cross section interference terms. In order to satisfy condition (23) we further choose

$$V_{LR}^f = V_{RR}^f = -V_{LL}^f = -V_{RL}^f. \quad (24)$$

Finally, we assume family universality and in view of the conflict between A_{LR} and \mathcal{P}_τ^{FB} we restrict the new physics to contribute to hadrons only,

$$V_{ij}^l = 0. \quad (25)$$

Using exclusively L3 data actually yields a lower value of χ^2 by about 0.7. The fit result on the two new parameters, however, are such that A_{LR} would actually be decreased. Including the SLC data then gives values for the new parameters very close to zero and hence we dismiss case (a) as uninteresting.

To maximize positive contributions to A_{LR} in case (b) we set

$$V_{RL}^f = V_{RR}^f = 0. \quad (26)$$

Interestingly, this suppresses contributions to forward-backward asymmetries and final state polarizations. Again we assume that condition (25) holds. Finally, motivated by the large fraction of b -events observed at LEP we set

$$\begin{aligned} V_{LL}^b &= -V_{LR}^b \neq 0, \\ V_{ij}^f &= 0 \quad \text{otherwise.} \end{aligned} \quad (27)$$

There are interferences in case (b), as well, but their g_A^e/g_V^e enhancement for A_{LR} no longer holds.

Using the LEP values $\sin^2 \theta_{\text{eff}}^e = 0.2321 \pm 0.0004$ and $\frac{\sigma_b^{\text{new}}}{\sigma_b} \approx 2\%$ and again neglecting the τ -events at SLC we find

$$A_{LR}^0 = (1 - \frac{\sigma_b^{\text{new}}}{\sigma_b} R_b) A_{LR}^{SM} + \frac{\sigma_b^{\text{new}}}{\sigma_b} R_b = 0.1462, \quad (28)$$

which is still far lower than the measured value. In fact, we would need a fraction $\frac{\sigma_{\text{had}}^{\text{new}}}{\sigma_{\text{had}}} \approx 2.5\%$ of hadrons from new physics to account for the SLD result. Given the fact that $\sigma_{\text{had}}^0 = 41.49 \pm 0.12$ [nb] and $R_l = \frac{\sigma_{\text{had}}^0}{\sigma_l^0} = 20.795 \pm 0.040$ are measured with 3 and 2 per mill accuracy, respectively, does not leave much room for achieving that. But as mentioned before, a new Z lineshape fit including the new operators might change the picture:

We allow the new physics to couple to all quark flavors in a family universal way ($V_{LL}^d = -V_{LR}^d = V^d$ and likewise for V^u). Using only L3 data we find $V^u = -0.9 \pm 1.4$ and $V^d = -0.5 \pm 0.9$ with a slight decrease in χ^2 (-0.3 compared to the Standard Model fit). Including the SLC data yields $V^u = -1.9 \pm 0.6$ and $V^d = -1.1 \pm 0.3$. Note, that the obtained values are consistent with the $SU(2)$ symmetric case, $V^u = V^d$. In such a fit we find for the lineshape parameters,

$$\begin{aligned} M_Z &= 91.184 \pm 0.012 \text{ GeV}, \\ \Gamma_Z &= 2.468 \pm 0.020 \text{ GeV}, \\ \sigma_{\text{had}}^0 &= 41.19 \pm 0.25 \text{ nb}, \\ \Gamma_l &= 82.74 \pm 0.64 \text{ MeV}. \end{aligned} \quad (29)$$

They deviate significantly from their Standard Model counterparts. In this case only about 21% of the A_{LR} discrepancy can be accounted for and still amounts to 2.5σ . The discrepancy is actually a little smaller than that because of the effect of the additional b -quarks on $A_{FB}(b)$ as discussed in the previous section. Although this scenario is interesting in that it describes the data better than the Standard Model, we find only a modest increase in A_{LR} . Actually, it turned out that these operators may rather account for the large observed partial Z width into b -quarks. The value of $V^d = -1.1$ corresponds to a fraction of about 1% b -quarks due to new physics. Cross sections into up-type quarks are rather lowered (which would also be consistent with observation), but not significantly so. These new operators may have an effect on low energy data as well. But since by demand they do not interfere with QED this effect is much smaller than in the pure vector case, and there would be an enhancement in the hadronic cross section. We do not use these low energy data to quantitatively constrain these operators, since there may be competing residual interferences or the new physics may actually decrease faster towards lower energies than is the case for a pure four-Fermi operator.

IV. CONCLUSIONS

We can finally conclude that $e^+ e^-$ data do not support the idea that the presence of new four-Fermi operators might solve the conflict between SLC and LEP asymmetry measurements. (Pseudo-) scalar and tensor operators do not give rise to polarization asymmetries. Forward-backward asymmetries are only possible when helicity conserving scalar and tensor interactions interfere. If the observed surplus of $Z \rightarrow b\bar{b}$ events is real and due to this class of operators the value of $\sin^2 \theta_{\text{eff}}^e$ extracted from $A_{FB}(b)$ may significantly change. This is important since presently $A_{FB}(b)$ serves as the most precise determination of the weak mixing angle.

(Axial-) vector operators are constrained to have basically no interference with the photon. We obtained this constrained by considering lower energy $e^+ e^-$ annihilation data and using it we showed that the interference effect with the Z cannot explain the large left-right asymmetry. If the new physics couples predominantly to left-handed electrons there is a sizable effect on A_{LR} , but less than a fourth of the discrepancy can be accounted for. Still this class of operators is interesting since (1) it may account for the surplus of observed $b\bar{b}$ final states; (2) it has significant impact on extracted Z pole parameters, in particular total and partial widths; and (3) it can account at least for part of the A_{LR} puzzle.

New (axial-) vector operators could for instance arise through a new Z' -boson. This case is discussed in a very recent paper by Caravaglios and Ross [12]. They conclude that only an (almost) degenerate Z' can affect the SLAC measurement while leaving the LEP observables unaffected. Indeed, a Z' resonating close to the Z is only poorly described by four-Fermi interactions since its amplitude falls off much faster at lower energies than is the case for contact operators. Moreover, it strongly interferes *at* the pole and not only *near* it. On the other hand, it should be possible to further constrain the strength of its couplings by considering its interference with the photon at lower energies. Although the pure photon exchange dominates at energy scales such as at PETRA, the Z already contributes significantly, mainly through its interference with QED. The size of the analogous effect of $Z' - \gamma$ mixing for a Z' such as considered in [12] depends on how it couples to fermions. There is, however, a relative enhancement of this effect as the Z' is described as predominantly vector-like. Thus it would be interesting to study its implications at lower energies.

In view of fit results such as (29) we must further conclude that in the presence of certain kinds of new physics our knowledge of Z boson properties may be poorer than expected. The generally low χ^2 values of Z lineshape fits should not be taken as a guarantee

that new physics contributions are necessarily negligible. Therefore we encourage the LEP collaborations to perform in addition to their usual “model independent fits” more general fits allowing new physics such as the kind discussed in this paper, in which we could only attempt a semi-quantitative discussion. We were forced to use a number of approximate treatments for our fits as described in appendix A. Despite of all these simplifications, our agreement with L3 is very good and these approximations turned out to be not very crucial. We believe that an incorporation of forward-backward asymmetries into the fits may be worthwhile and to this end we encourage experimenters to systematically present raw data in addition to extracted or bare quantities. There is another benefit to it: the extraction of bare quantities often requires at some point assumptions such as the validity of the Standard Model when data from outside are input. This usually leads to small effects, but it destroys the consistency of global fits to high precision data.

ACKNOWLEDGEMENTS

It is a pleasure to thank Paul Langacker for his suggestion to study the topic presented in this article, for his careful reading of the manuscript and for numerous very helpful suggestions and comments. This work was supported by the Texas National Laboratory Research Commission, by the D.O.E. under contract DE-AC02-76-ERO-3071 and by the Deutsche Forschungsgemeinschaft.

APPENDIX A: APPROXIMATIONS FOR LINESHAPE FITS

In this appendix we list the simplifications we used for our fits:

1. We worked in the improved Born approximation, i.e., box contributions were neglected and the s -dependent effective couplings were assigned their values at $\sqrt{s} = M_Z$.
2. In the Z interference terms we kept the effective mixing angles fixed. The axial-vector couplings are absorbed into the widths (see appendix B).
3. For hadronic cross sections we neglected systematic uncertainties from selection cuts, efficiencies and backgrounds, which are small compared to the luminosity error.
4. In order to avoid the use of a high dimension correlation matrix we proceeded in the following way: We defined a luminosity scale factor by

$$L = \frac{\frac{1}{(\Delta L)^2} + \sum \frac{\sigma_{\text{had}}^{\text{fit}} \sigma_{\text{had}}^{\text{exp}}}{(\Delta \sigma_{\text{had}}^{\text{exp}})^2}}{\frac{1}{(\Delta L)^2} + \sum \frac{\sigma_{\text{had}}^{\text{fit}^2}}{(\Delta \sigma_{\text{had}}^{\text{exp}})^2}}, \quad (\text{A1})$$

where the sum is over the scan points. We then substituted for hadrons *and* leptons $\sigma^{\text{fit}} \rightarrow L\sigma^{\text{fit}}$ in our χ^2 function and added the term

$$\frac{(L - 1)^2}{(\Delta L)^2}, \quad (\text{A2})$$

where the luminosity error $\Delta L = 0.006$ is taken to be fully correlated between the different years. For $\Delta L \rightarrow \infty$, i.e., disregarding the constraint from the direct luminosity measurement, this procedure corresponds to a determination of the luminosity from the cross section data.

5. We chose a similar treatment for the systematic uncertainties (excluding the luminosity error) for μ and τ final states, which we assumed to be mutually uncorrelated. Again, these uncertainties were taken to be completely correlated between different years and equal, although the 1990 errors were slightly higher than the ones from 1991 and 1992.
6. Initial state radiation is included in one loop exponentiated form except for the pure QED contribution where the exponentiation can be omitted [13]. We present the explicit formulae including four-Fermi terms and their interferences in appendix B.
7. Some convolution integrals must be extended to a larger region if analytical formulae are desired [14].

8. We did not include $e^+ e^- \rightarrow e^+ e^-$ data to avoid complications due to t -channel exchange.
9. We did not include any forward-backward asymmetry data. Their inclusion is straightforward but somewhat tedious, since another class of convolution integrals has to be calculated.
10. We did not include total cross section data of 1993, since they are not yet available in published form.

It turned out a posteriori that these simplifications have little impact on the fit results. If we compare our Standard Model fit with the one by the L3 collaboration⁸ [8],

our fit	L3	
$M_Z [\text{GeV}] = 91.193 \pm 0.006$	$91.195 \pm 0.006 \pm 0.007 (\text{LEP}),$	
$\Gamma_Z [\text{GeV}] = 2.496 \pm 0.010$	$2.494 \pm 0.009 \pm 0.005 (\text{LEP}),$	
$\sigma_0 [\text{nb}] = 41.42 \pm 0.20$	$41.41 \pm 0.26,$	
$\Gamma_l [\text{MeV}] = 83.62 \pm 0.35$	$83.55 \pm 0.60,$	
$\chi^2 = 41/44$	$53/60,$	(A3)

we see that the central values agree within better than 0.1%. We reached this precision without any numerical integration or matrix manipulation so that the fit runs are very fast. The precision can be further improved, if desired.

APPENDIX B: INITIAL STATE RADIATION

We treated initial state radiation using the approximations described in reference [13]. Before convolution, the total cross section formula reads

$$\sigma'_f(s) = \left(\frac{s^2 C_{I4F} + s(C_R + C_I - M_Z^2 C_{I4F}) - M_Z^2 C_I}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} + \frac{C_Q}{s} + s C_{4F} + C_{IQ4F} \right) (1 + \frac{3}{4\pi} \alpha(M_Z) Q_f^2) (1 + \delta_{QCD}) \sqrt{1 - 4m_f^2/M_Z^2}, \quad (\text{B1})$$

where for instance for a pure vector type four-Fermi contribution

⁸The quoted L3 result of Γ_l is the weighted average of Γ_μ and Γ_τ , whereas the χ^2 value includes $e^+ e^-$ final states, as well. Our quoted errors are those returned by the minimization routine MINUIT.

$$\begin{aligned}
C_R &= \frac{12\pi\Gamma_e\Gamma_f}{M_Z^2+\Gamma_Z^2}, \\
C_Q &= \frac{4}{3}\pi Q_f^2\alpha(M_Z)^2 N_c, \\
C_{4F} &= \frac{4\pi N_c V_f^2}{3\Lambda^4}, \\
C_I &= \frac{8\pi\alpha(M_Z)Q_e Q_f M_Z \sqrt{N_c\Gamma_e\Gamma_f}}{M_Z^2+\Gamma_Z^2} \frac{g_V^e}{\sqrt{g_V^{e^2}+1}} \frac{g_V^f}{\sqrt{g_V^{f^2}+1}}, \\
C_{I4F} &= \frac{8\pi V_f M_Z \sqrt{N_c\Gamma_e\Gamma_f}}{\Lambda^2(M_Z^2+\Gamma_Z^2)} \frac{g_V^e}{\sqrt{g_V^{e^2}+1}} \frac{g_V^f}{\sqrt{g_V^{f^2}+1}}, \\
C_{I4FQ} &= \frac{8\pi Q_e Q_f \alpha(M_Z) V_f N_c}{3\Lambda^2}, \\
\tilde{M}_Z &= \frac{M_Z}{\sqrt{1+\Gamma_Z^2/M_Z^2}}, \\
\tilde{\Gamma}_Z &= \frac{\Gamma_Z}{\sqrt{1+\Gamma_Z^2/M_Z^2}}.
\end{aligned} \tag{B2}$$

The partial widths are with QED, QCD and mass corrections removed; g_V^f denotes the vector coupling of fermion f ; and δ_{QCD} is the QCD correction, which in case of b -quarks is m_b and m_t -dependent [15] and a weighted average of the corrections to the vector and axial-vector partial widths has to be used.

Initial state radiation is included by computing the convolution integral

$$\sigma_f(s) = \int_0^{1-\frac{s_0}{s}} dx \sigma'_f[s(1-x)] G(x), \tag{B3}$$

where s_0 is taken to be $4m_f^2$ for leptons and $(10 \text{ GeV})^2$ for hadrons. $G(x)$ is the radiator function; to one-loop exponentiated approximation it is given by [16]

$$G(x) = \beta x^{\beta-1} \delta^{V+S} - \frac{\alpha}{\pi} (2-x)(L-1), \tag{B4}$$

where

$$\begin{aligned}
L &= \ln \frac{s}{m_e^2}, \\
\beta &= \frac{2\alpha}{\pi} (L-1), \\
\delta^{V+S} &= 1 + \frac{\alpha}{\pi} \left(\frac{3}{2}L + \frac{\pi^2}{3} - 2 \right).
\end{aligned} \tag{B5}$$

The final result is

$$\begin{aligned}
\sigma_f(s) &= \left\{ (C_{I4F} + \frac{C_R+C_I-M_Z^2 C_{I4F}}{s} - \frac{M_Z^2 C_I}{s^2}) (J_\beta \delta^{V+S} - \frac{\beta}{2} J_1) \right. \\
&\quad - (2C_{I4F} + \frac{C_R+C_I-M_Z^2 C_{I4F}}{s}) (\frac{\beta}{\beta+1} J_{\beta+1} \delta^{V+S} - \frac{\beta}{2} J_2) \\
&\quad \left. + C_{I4F} (\frac{\beta}{\beta+2} J_{\beta+2} \delta^{V+S} - \frac{\beta}{2} J_3) \right. \\
&\quad + \frac{C_Q}{s} [1 + \frac{\alpha}{\pi} L \left(\frac{1}{2} + \frac{s_0}{s} - \ln \frac{s_0}{s} + 2 \ln(1 - \frac{s_0}{s}) \right) + \frac{\alpha}{\pi} \left(\frac{\pi^2}{3} - 1 - \frac{s_0}{s} + \ln \frac{s_0}{s} - 2 \ln(1 - \frac{s_0}{s}) \right)] \\
&\quad + s C_{4F} \left[\frac{\beta \frac{s_0}{s} + 1}{\beta+1} (1 - \frac{s_0}{s})^\beta \delta^{V+S} - \frac{\beta}{2} \left(\frac{5}{6} - \frac{s_0^2}{2s^2} - \frac{s_0^3}{3s^3} \right) \right] \\
&\quad \left. + C_{I4FQ} \left[(1 - \frac{s_0}{s})^\beta \delta^{V+S} - \frac{\beta}{2} \left(\frac{3}{2} - \frac{s_0}{s} - \frac{s_0^2}{2s^2} \right) \right] \right\} \\
&\quad (1 + \frac{3}{4\pi} \alpha(M_Z) Q_f^2) (1 + \delta_{QCD}) \sqrt{1 - 4m_f^2/M_Z^2}.
\end{aligned} \tag{B6}$$

In order to compute the J_n we extend the integration region to $s_0 = 0$ [13],

$$J_n = \int_0^1 dx \frac{x^{n-1}(2-x)}{(x+a)^2 + b^2}, \quad (\text{B7})$$

and we use the abbreviations

$$\begin{aligned} a &= \frac{\tilde{M}_Z^2}{s} - 1, \\ b &= \frac{\tilde{M}\tilde{\Gamma}_Z}{s}. \end{aligned} \quad (\text{B8})$$

The result is

$$\begin{aligned} J_1 &= \frac{2+a}{b}A - \frac{B}{2}, \\ J_2 &= \frac{a^2+2a-b^2}{b}A + (1+a)B - 1, \\ J_3 &= \frac{2a^2+a^3-2b^2-3ab^2}{b}A + \frac{b^2-3a^2-4a}{2}B + 2a + \frac{3}{2}, \end{aligned} \quad (\text{B9})$$

with

$$\begin{aligned} A &= \arctan \frac{a+1}{b} - \arctan \frac{a}{b}, \\ B &= \ln \frac{a^2+b^2+2a+1}{a^2+b^2}. \end{aligned} \quad (\text{B10})$$

The integration region for J_β with $\beta < 2$ is extended even further [13,14],

$$J_\beta = \beta \int_0^\infty dx \frac{x^{\beta-1}}{x^2 - 2\eta x \cos \zeta + \eta^2} = \eta^{\beta-2} \frac{\pi \beta \sin[(1-\beta)\zeta]}{\sin \pi \beta \sin \zeta}, \quad (\text{B11})$$

and we defined

$$\begin{aligned} \eta &= \sqrt{a^2 + b^2}, \\ \cos \zeta &= \frac{a}{\eta}. \end{aligned} \quad (\text{B12})$$

We checked that the extension of the integration domain can be understood as an expansion in η (which for LEP energies is of order $\mathcal{O}(\Gamma_Z/M_Z)$) and keeping only negative powers. $J_{\beta+2}$ has no negative powers of η and we have to keep the η independent term to insure that it vanishes in the limit $\beta \rightarrow 0$. We obtain ($0 \leq \beta < 1$),

$$J_{\beta+2} = \frac{\beta+2}{\beta} [1 - \eta^2 J_\beta - 2 \cos \zeta \frac{\beta}{\beta+1} \eta J_{\beta+1}]. \quad (\text{B13})$$

Actually, in case of J_β above, one should for the sake of self-consistency keep at least the η -independent term as well, since it is comparable to the J_n terms. We find up to terms linear in η ($\beta < 3$),

$$J_\beta = \beta \left[\eta^{\beta-2} \frac{\pi \sin[(1-\beta)\zeta]}{\sin \pi \beta \sin \zeta} + \frac{1}{\beta-2} - \frac{2}{\beta-3} \eta \cos \zeta + \mathcal{O}(\eta^2) \right]. \quad (\text{B14})$$

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